

Q.P. Code – 42534

**Fifth Semester B.Sc. Degree Examination,
October/November 2019**

(CBCS Semester Scheme)

Mathematics

Paper 5.2 (A) – ANALYSIS AND INTEGRAL TRANSFORMS

Time : 3 Hours]

[Max. Marks : 90

Instructions to Candidates :

1. Answers ALL questions.
2. Answer should be written completely in English.

PART – A

I. Answer any **SIX** questions :

(6 × 2 = 12)

1. Evaluate : $\int_0^{\infty} x^2 e^{-x^2} dx$ using Gamma function.

2. Evaluate $\beta(5.5, 3)$.

3. Define half-range cosine series in $(0, L)$.

4. Evaluate : $L\{\sin^2 t\}$.

5. Evaluate : $L^{-1}\left\{\frac{1}{S(S^2+3)}\right\}$.

6. Define inverse Fourier sine transform of $F_s(\alpha)$.

7. Prove that $F[f(ax)] = \frac{1}{a} F\left(\frac{u}{a}\right)$.

PART – B

II. Answer any **SIX** questions :

(6 × 3 = 18)

8. Show that $\beta(n, m) = \beta(m, n)$.

9. Evaluate $\int_0^{\infty} \frac{x^6(1-x^8)}{(1+x)^{22}} dx$.

10. State Dirichlet's condition.



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11. Find $L\{F(t)\}$ if $F(t) = \begin{cases} 0, & 0 \leq t \leq 4 \\ 2t, & t \geq 4 \end{cases}$.

12. Find $L\{t \cos t\}$.

13. Prove that $f(x) = \frac{1}{\pi} \int_0^{\infty} \int_{-\infty}^{\infty} f(t) \cos(\mu(t-x)) dt du$.

14. Prove that $F_s[f'(x)] = -\alpha F_c[f(x)]$.

PART - C

III. Answer any **FOUR** questions :

(4 × 5 = 20)

15. With the usual notation prove that $\sqrt{(1/2)} = \sqrt{\pi}$.

16. Using Beta and Gamma function, evaluate $\int_0^1 \frac{x^2}{\sqrt{1-x^4}} dx \times \int_0^1 \frac{dx}{\sqrt{1+x^4}}$.

17. Obtain Fourier series for $f(x) = \begin{cases} 1+2x & -3 \leq x \leq 0 \\ 1-2x & 0 \leq x \leq 3 \end{cases}$ in $(-3, 3)$.

18. Obtain the Fourier series for $f(x) = e^{-ax}$ in $(0, 2\pi)$.

19. Obtain the Fourier half range sine series for $f(x) = x(\pi-x)$ over $(0, \pi)$. Hence deduce that $\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$.

IV. Answer any **FOUR** questions :

(4 × 5 = 20)

20. If $L\{f(t)\} = F(s)$, then prove that

(a) $L\{e^{at} f(t)\} = F(s-a)$

(b) $L\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$.

21. Evaluate :

(a) $L\{\sin 3t \cos 2t\}$

(b) $L\{(t+2)e^{-5t}\}$.

22. Evaluate $L^{-1}\left\{\frac{s+8}{s^2+4s+6}\right\}$.

23. Evaluate $L^{-1}\left\{\frac{1}{s(s^2+9)}\right\}$ using convolution theorem.

24. Solve $y'' + 4y = 3$, given $y(0) = 0 = y'(0)$ using Laplace transforms.

V. Answer any **FOUR** questions :

(4 × 5 = 20)

25. Find the Fourier integral expansion of $f(x) = \begin{cases} 1 & \text{for } 0 \leq x \leq \pi \\ 0 & \text{for } x > \pi \end{cases}$. Hence evaluate

$$\int_0^{\infty} \left(\frac{1 - \cos \pi u}{u} \right) \sin ux \, dx.$$

26. Obtain the Fourier transform of $f(x) = e^{-a^2 x^2}$, where a is a constant.

27. Find the function $f(x)$ whose cosine transform is given by

$$F_c[f(x)] = \begin{cases} a - \frac{\alpha}{2}, & 0 \leq \alpha \leq 2a \\ 0, & \alpha \geq 2a \end{cases}$$

28. Show that $x e^{\frac{-x^2}{2}}$ is self reciprocal w.r.to Fourier sine transformation.

29. Find the finite Fourier cosine transform of $f(x) = \sin nx$.

